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Introduction to Geophysical Modelling and Inversion

James Reid

GEOPHYSICAL INVERSION FOR MINERAL EXPLORERS

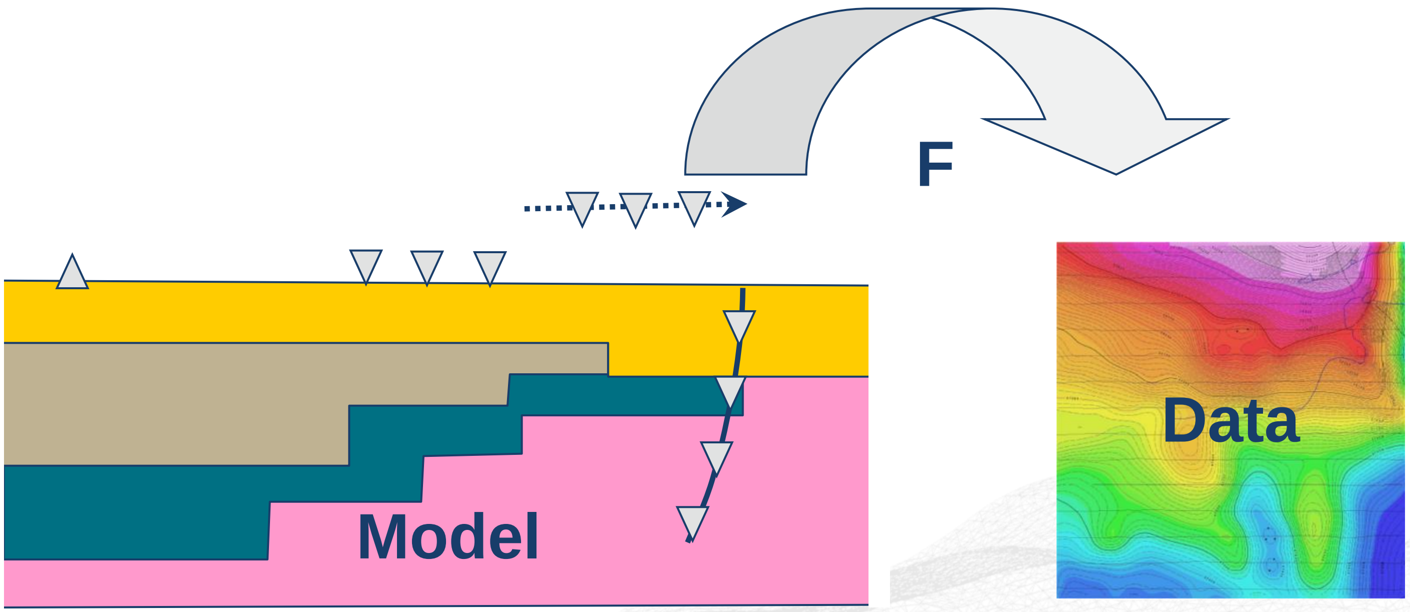
ASEG-WA, SEPTEMBER 2014

Forward modelling vs. inversion

Forward Modelling: Given a model **m** and predicting data **d**

$$\mathbf{d=F(m)}$$

F is an operator representing the governing equations relating the model and data



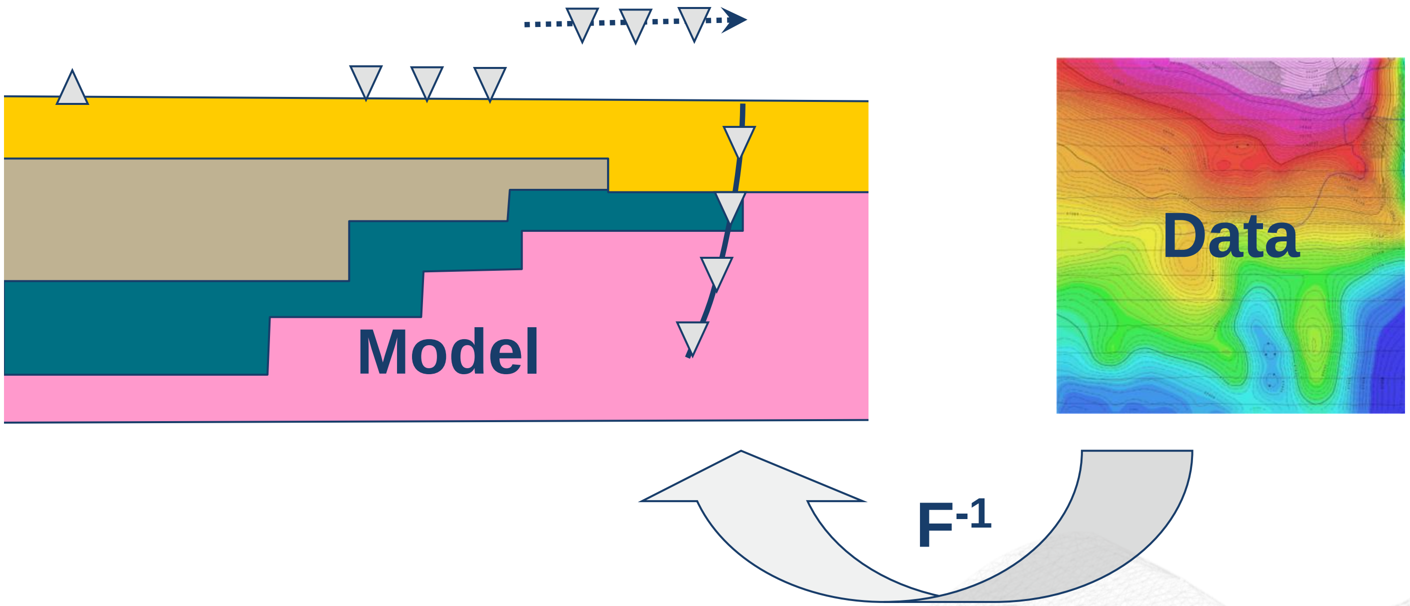
Inversion

Geophysical inversion refers to the mathematical and statistical techniques for recovering information on subsurface physical properties (magnetic susceptibility, density, electrical conductivity etc) from observed geophysical data.

What is inversion?

Inversion: Recording data **d** and predicting model **m**

$$\mathbf{m=F^{-1}(d)}$$



What is inversion?

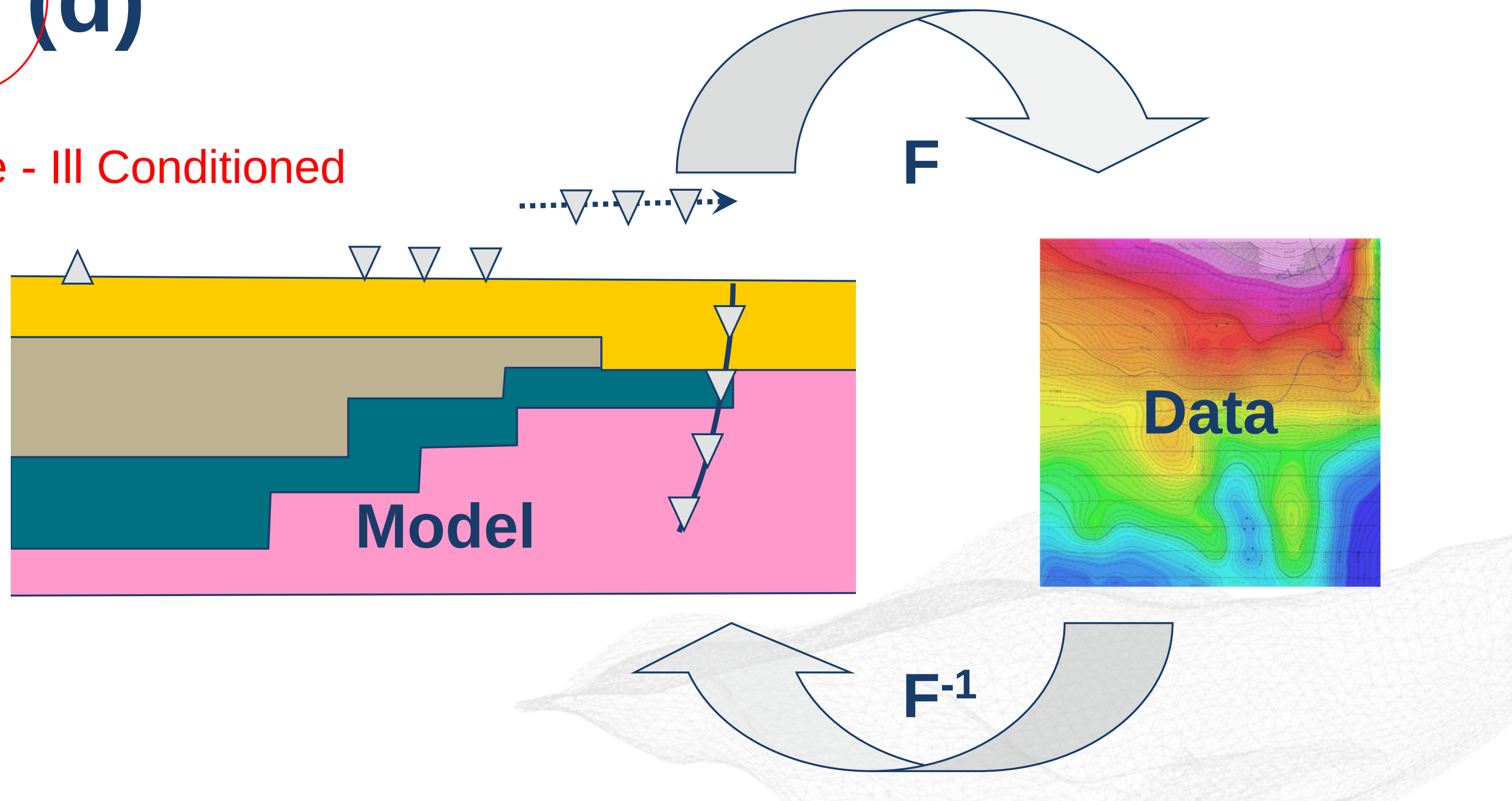
Forward Modelling: Given a model and predicting data

$$d=F(m)$$

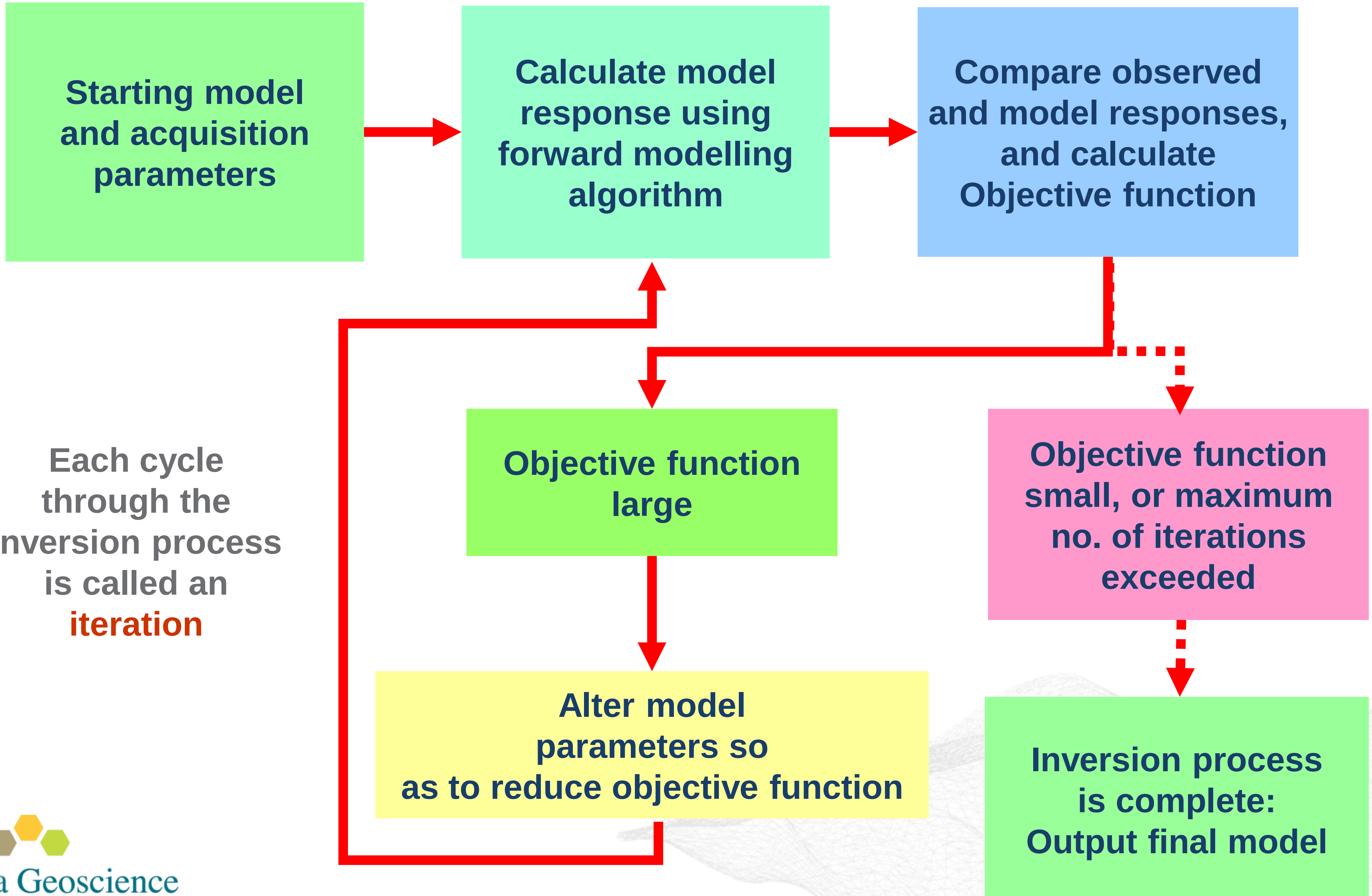
Inversion: Recording data and predicting model

$$m=F^{-1}(d)$$

Not Possible - Ill Conditioned



Iterative inversion



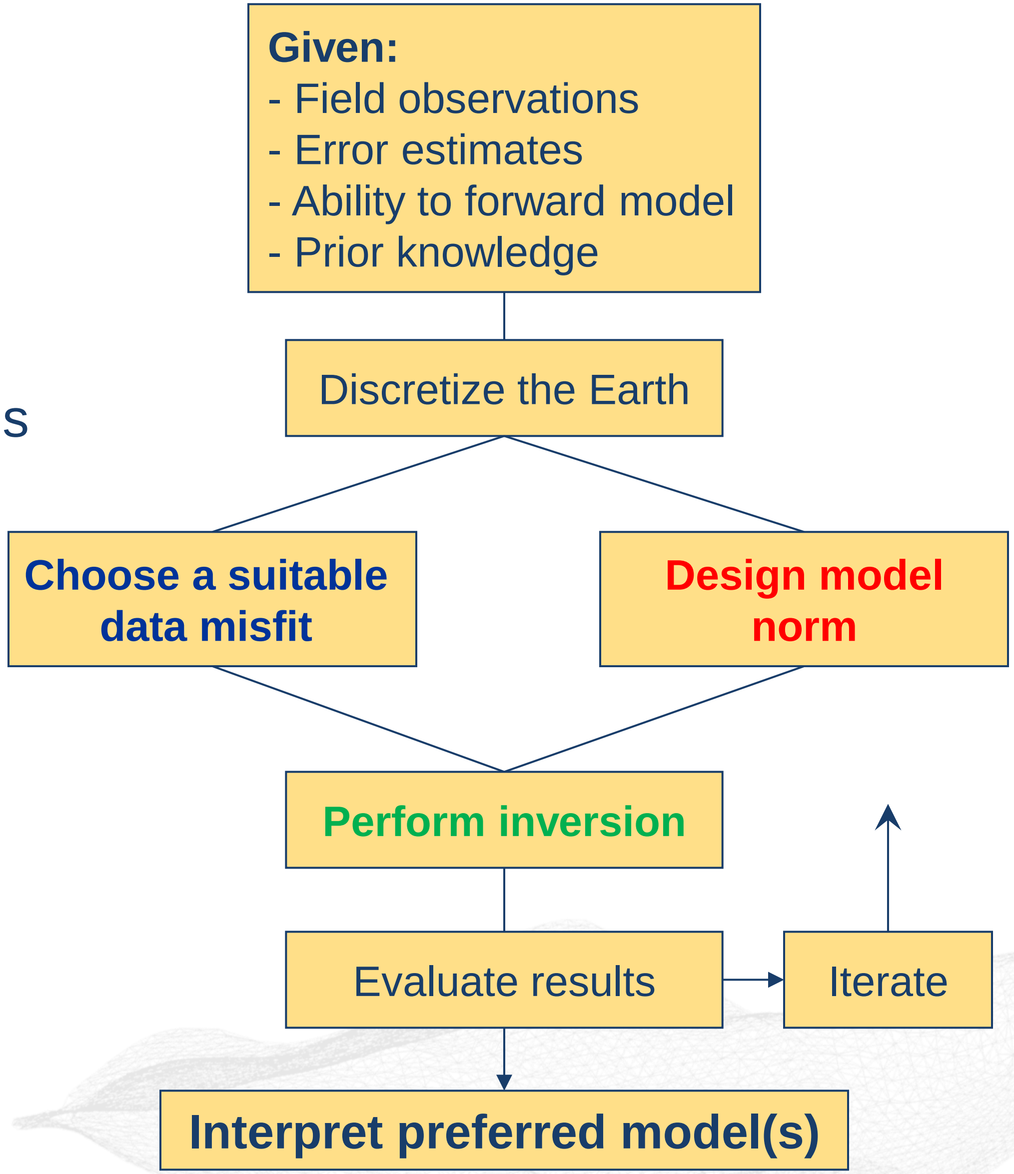
Each cycle through the inversion process is called an **iteration**

How do inversions work?

This chart summarizes the requirements for proceeding with inversion of geophysical data.

Each box has important implications for successful inversion.

Ability to do forward modelling calculations is assumed.



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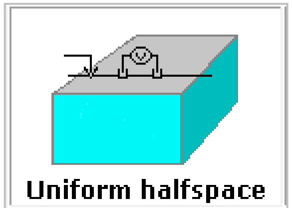
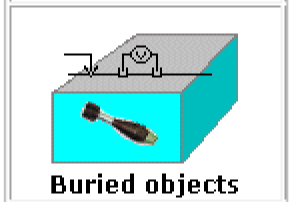
Models

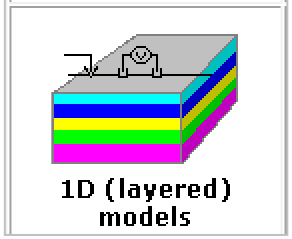
Model Types



Single Physical Property Value



Parameterized object (susceptibility, length, depth, orientation)



Physical property varies as a function of depth

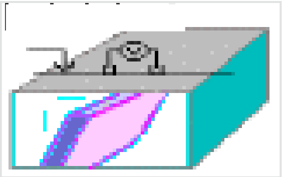


Plate in a free-space (vacuum)

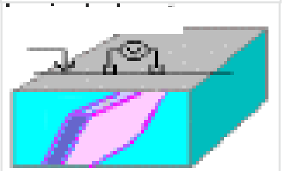


Plate in a half space

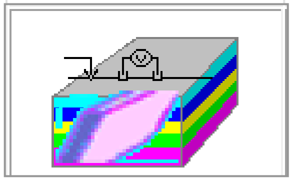
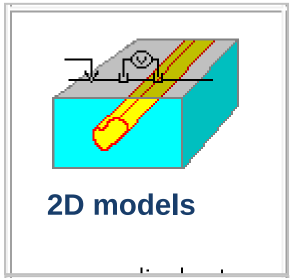


Plate in a layered model

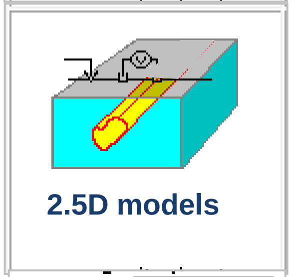
(after: Inversion for Applied Geophysics)

Models

Model Types

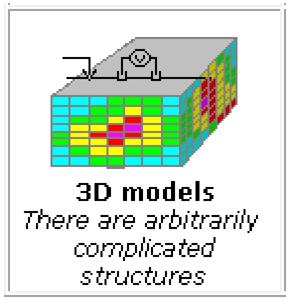


Model is unchanging perpendicular to profile section

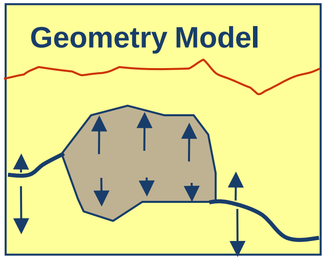


Model objects have limited strike length

Concatenated 1D models



Physical properties change in all 3 directions.
Generalized structure



Geologic unit boundaries adjust location to create 3D shapes and bodies.

(after: Inversion for Applied Geophysics)

Why invert data?

Helps explain complex data sets

- e.g. DCIP, Gravity Gradiometry, AEM, ZTEM, DHEM
- Removes topography effects

Explains the data with a model(s) of the earth:

- Provides a quantitative model that can be analysed
- →What is the depth, geometry, volume, physical property of the model features?
- More easily relates to geology - easier for interpretation
- → What geologic features can be determined in the model?

Can QC the data, identify problematic data

- acquisition problems

Helps separate the noise from the signal in the data

- estimates the noise levels
- estimates depth of penetration

Example: Target in presence of geological noise

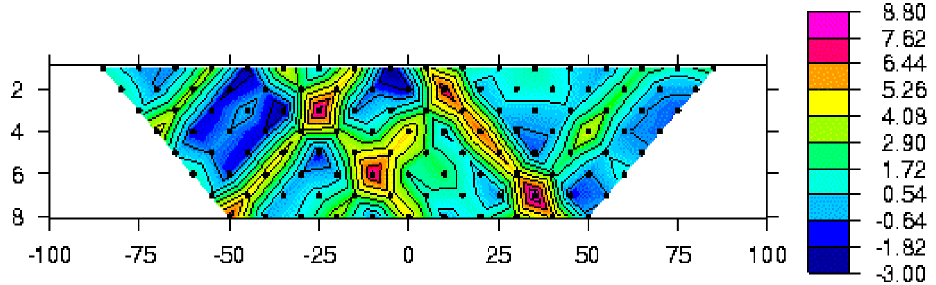
Data are sometimes difficult to interpret

Shallow anomalies represent chargeable boulders in till

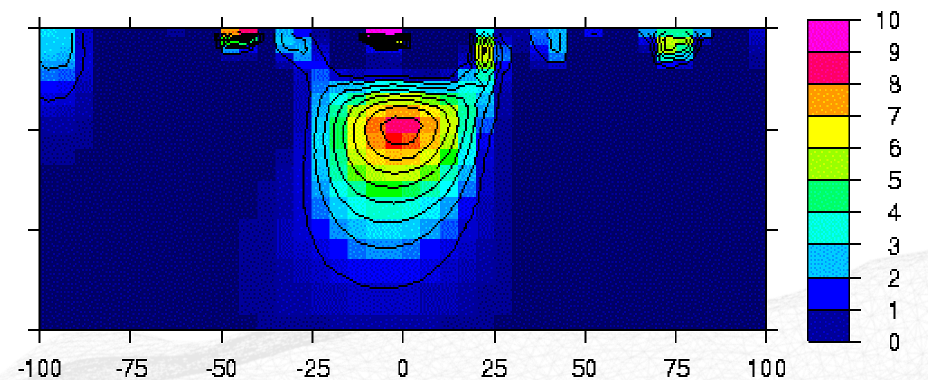
Subtle responses are important

Inversion result is more easily interpretable in terms of geology

IP data



Recovered chargeability



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Know The Data

In order for modelling to occur, all instrument system and survey acquisition parameters have to be known.

In general, try to do as little as possible to the data to preserve the information

Obviously erroneous data should be removed prior to inversion.

> This includes features/anomalies in the data which are not modelled by the forward modelling algorithm e.g., IP or SPM effects in EM data etc

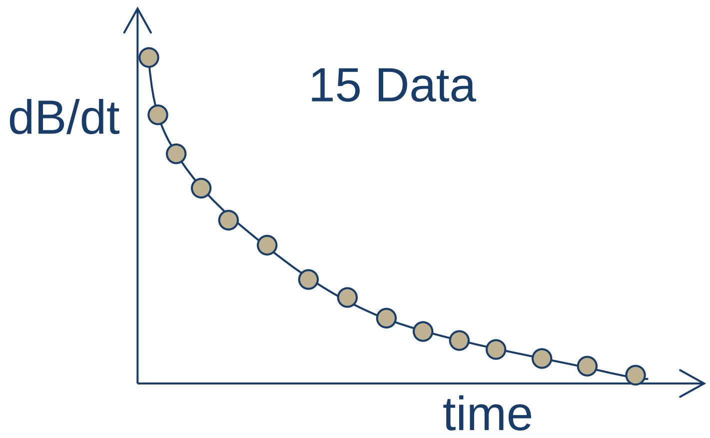
RUBBISH IN = RUBBISH OUT

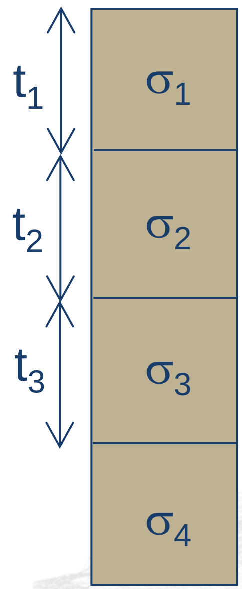
Inverse Modelling

Modelling Styles

- Parametric – few unknowns

e.g. TEM decay





1D Conductivity model

7 unknown
model parameters

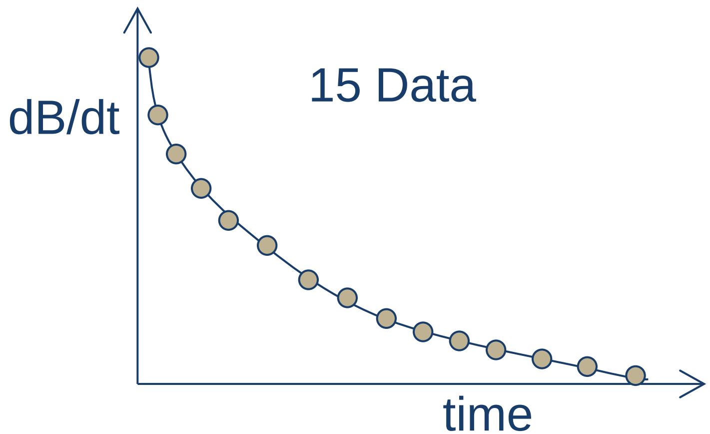
(conductivity of each layer;
thickness of upper three layers)

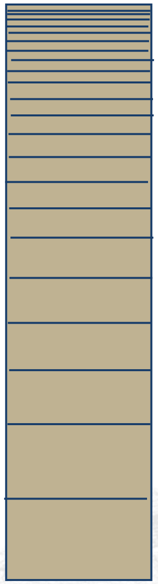
Inverse Modelling

Modelling Styles

- Parametric – few unknowns
- Generalized – many unknowns

e.g. TEM decay





1D Conductivity model

40 unknown
model parameters

1D Mesh structure predefined but smaller than expected structure of geology.

→ structure inferred from the resulting model

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Inverse Modelling

Modelling Styles

- Lithology based
  - VP suite (Fullagar Geophysics)
  - Geomodeller (Intrepid)
- Physical Property based
  - UBC-GIF codes
  - Geosoft Voxi
  - VP suite

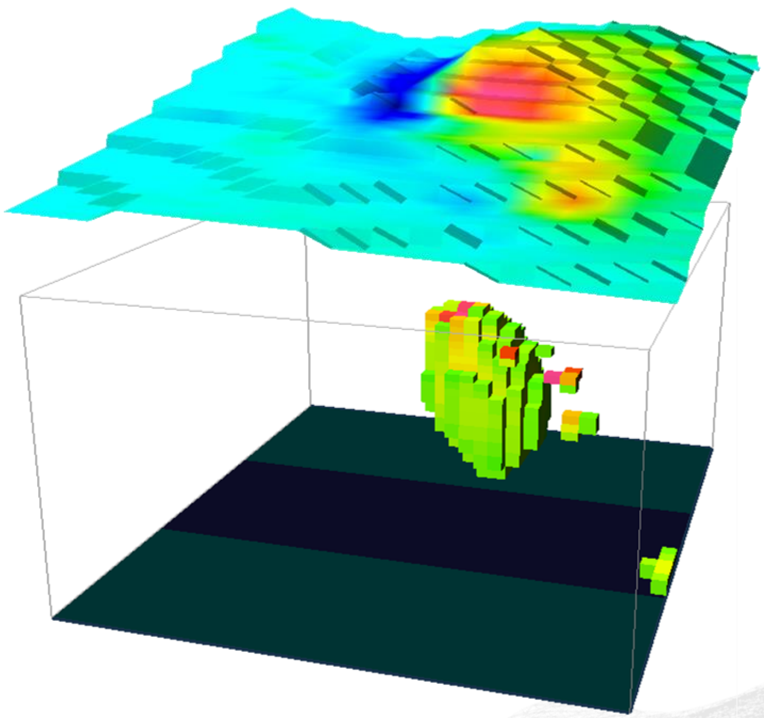
Inverse Modelling

Physical Property Based Modelling

- Physical property values of many individual cells are adjusted.
- General structure is recovered

e.g. Magnetic Data

3D susceptibility model



(low value cells removed)

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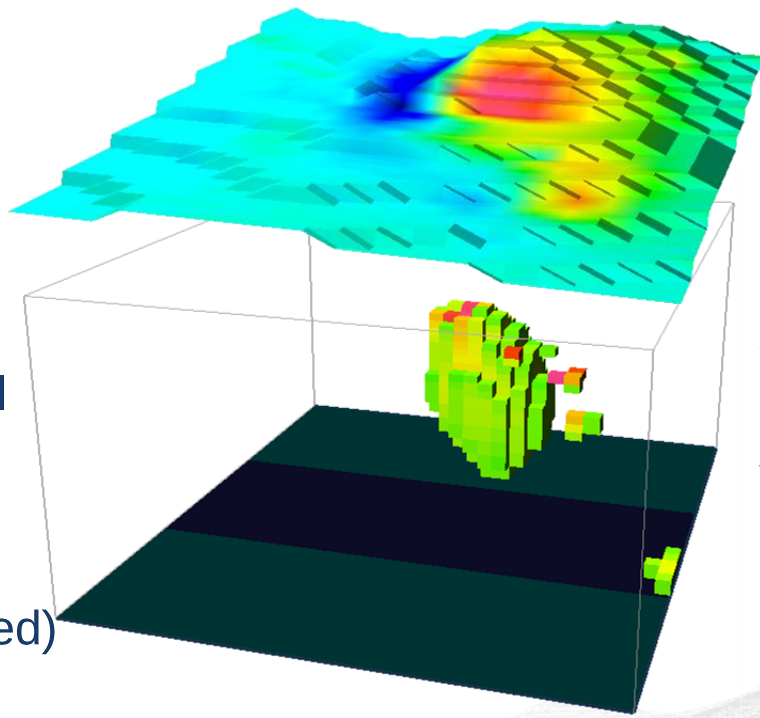
Inverse Modelling

Physical Property Based Modelling

- Physical property values of many individual cells are adjusted.
- General structure is recovered.



e.g. Magnetic Data

3D susceptibility model

(low value cells removed)

10,000+ unknown model parameters

3D Mesh structure predefined but smaller than expected structure of geology.

→ structure inferred from the resulting model

RESULT IS A PHYSICAL PROPERTY MODEL CONTAINING STRUCTURE

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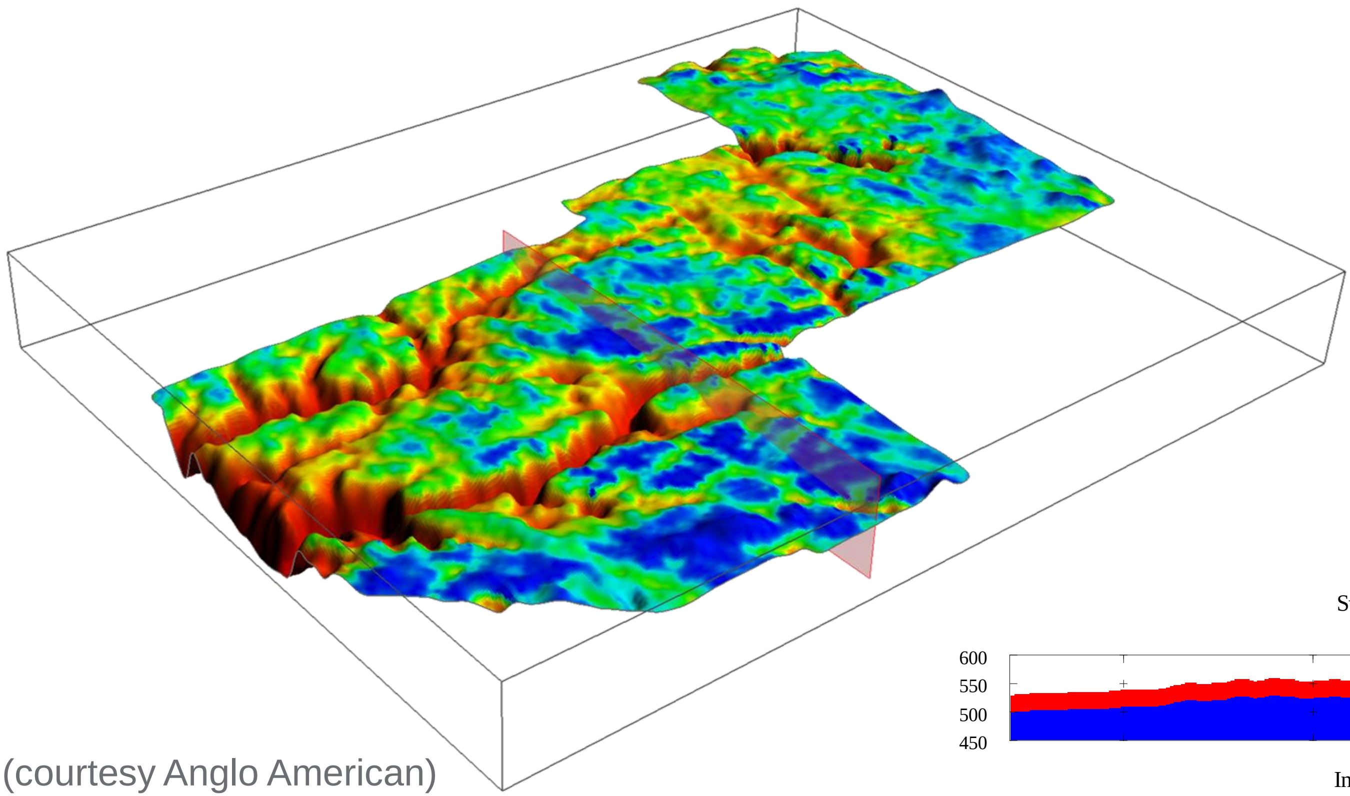
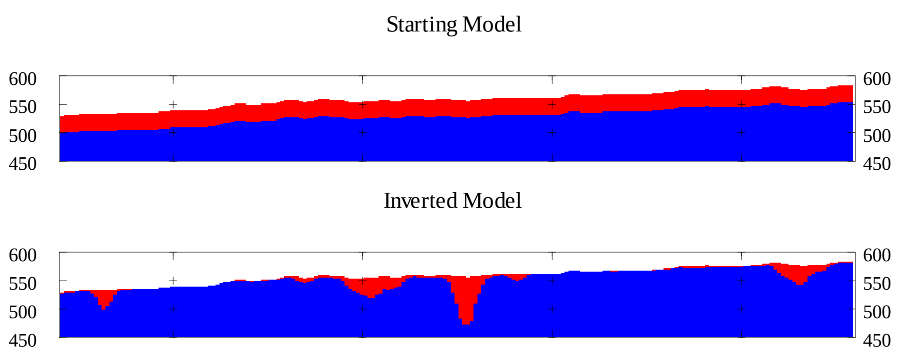
Inverse Modelling

Lithology Based Modelling

- Provide physical properties (single value or distribution) for each lithology and adjust the geometry to fit the data.



RESULT IS A GEOLOGICAL MODEL



(courtesy Anglo American)

Inverse Modelling

Which Modelling Style to choose?

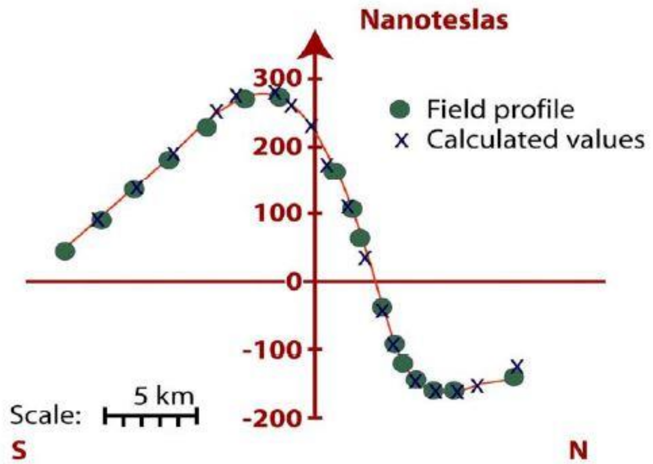
- Depends on the geophysical method, the survey design, and the exploration goal. Some examples might be:
  - Is the goal to define the geometry/volume?
    Measure the physical properties well and choose a lithologic based inversion (e.g. VPmg)
  - Is the goal to define a thickness of cover from a few TEM soundings?
    Use a parametric inversion
  - Is the goal to define both physical properties and geometry?
    Use a generalized inversion (e.g. UBC)
  - What geologic information is available that can be integrated into the modelling?

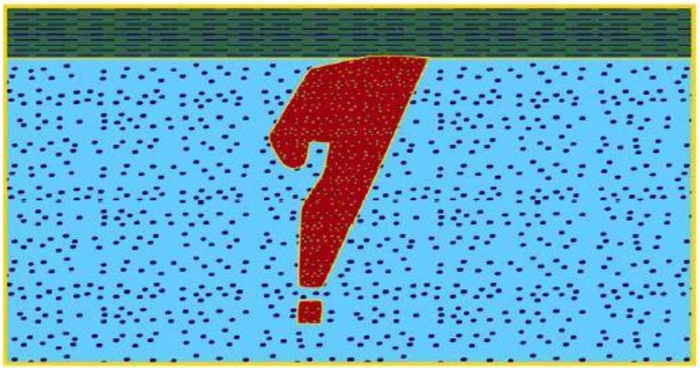
Acceptable models and non-uniqueness

There are **infinitely many** models that can explain the observed data

Why is this so?

- Because there are usually more unknowns (model parameters) than observed data points (underdetermined problem)
- Some physically-based non-uniqueness
- Real data contain noise





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Acceptable models and non-uniqueness

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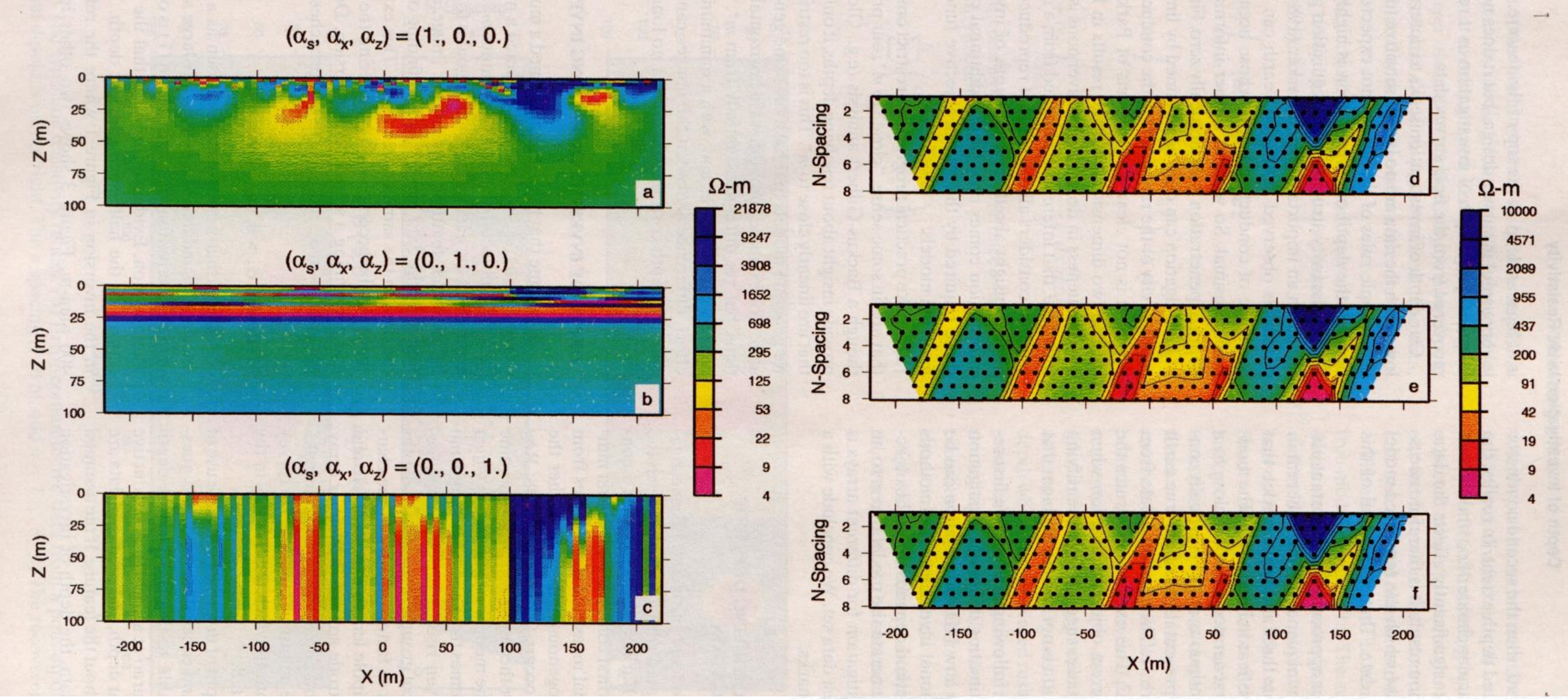

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How to chose one of infinitely many solutions?

Narrow down the range of options using prior knowledge

Geophysical prior knowledge:

- Values are positive, and/or within bounds
- Physical Properties: Estimates for host rock properties
- Point-location values from drill hole information

Logical prior knowledge:

- Find a “simple” result - as featureless as possible.
- This sacrifices resolution but prevents over-interpreting the data.

Geological prior knowledge:

- Character of the model (smooth, discontinuous)
- Some idea of scale length (or size) of the bodies
- Structural Constraints

Model norm

The model norm is a measure of the (mathematical) “size” of a model

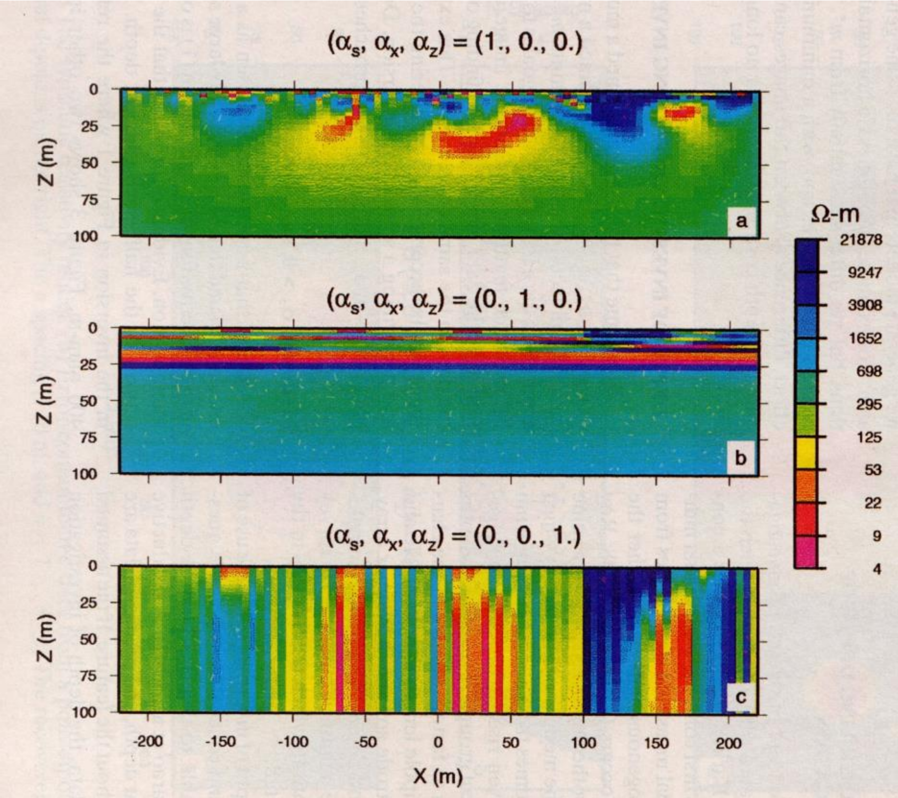
The inversion process is an automated decision making scheme

The model norm is a way of encoding prior information in a form suitable for mathematical optimisation – we seek the “smallest” model

The model norm is **part** of the solution to nonuniqueness …

Nonuniqueness is addressed by choosing the one model (from infinitely many) that minimizes the defined model norm

Model norms



Smooth

Minimum horizontal structure

Minimum vertical structure

Minimise difference between the model and some “reference model”

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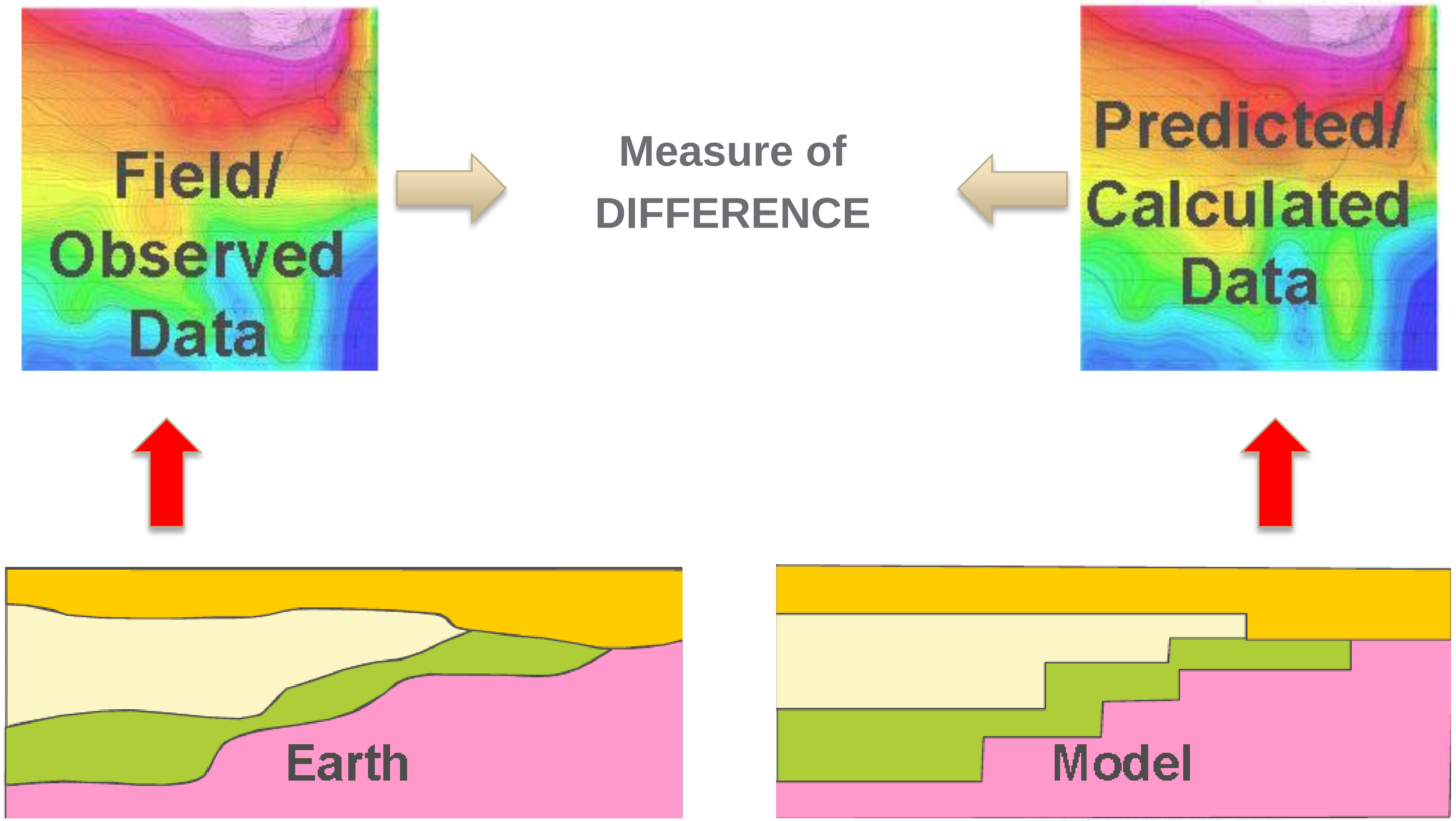
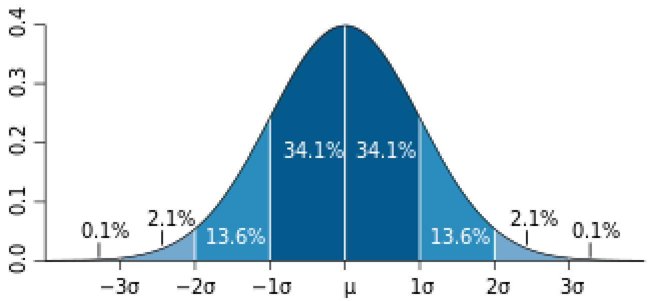
Data misfit



Field/
Observed
Data
Measure of
DIFFERENCE
Predicted/
Calculated
Data
Earth
Model

What is a good measure of misfit?

If we assume errors follow a particular distribution



then a measure of total misfit between predictions and field data can be defined $\phi_d(m)$:

- Predictions can be considered OK when
  $\phi_d(m) < tolerance$
- We don't want to fit the data too closely or we are fitting noise
- Not all measures of Data Misfit are equal

What contributes to data noise?

Natural and cultural noise sources

Accuracy and precision in data measurements

Data positioning errors

Approximations made in forward modelling

- 1D
- 2D
- 3D
- Plates
- Anisotropy
- Discretization of topography

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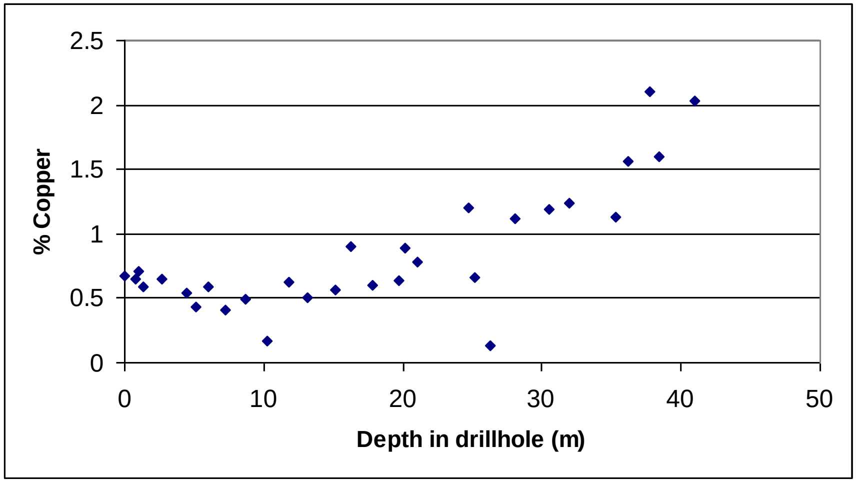
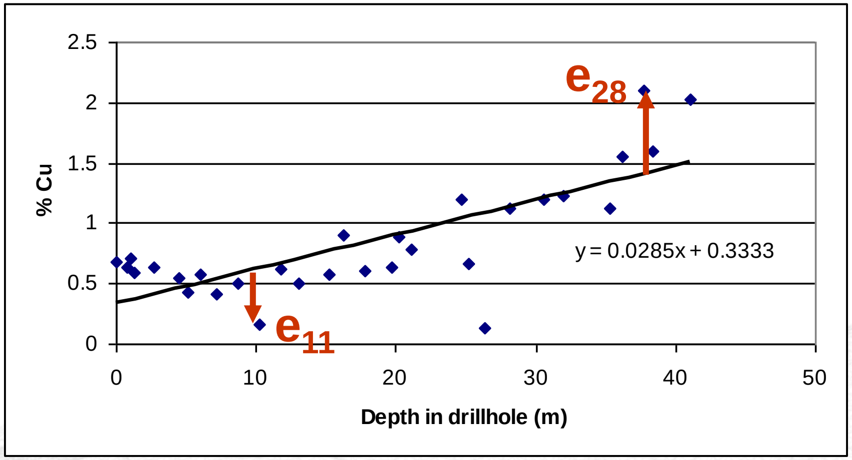
Measures of misfit

Consider the problem of fitting a straight line to the data shown below:





The residuals are the differences between the data points and the best-fit line at each depth

They may be positive or negative

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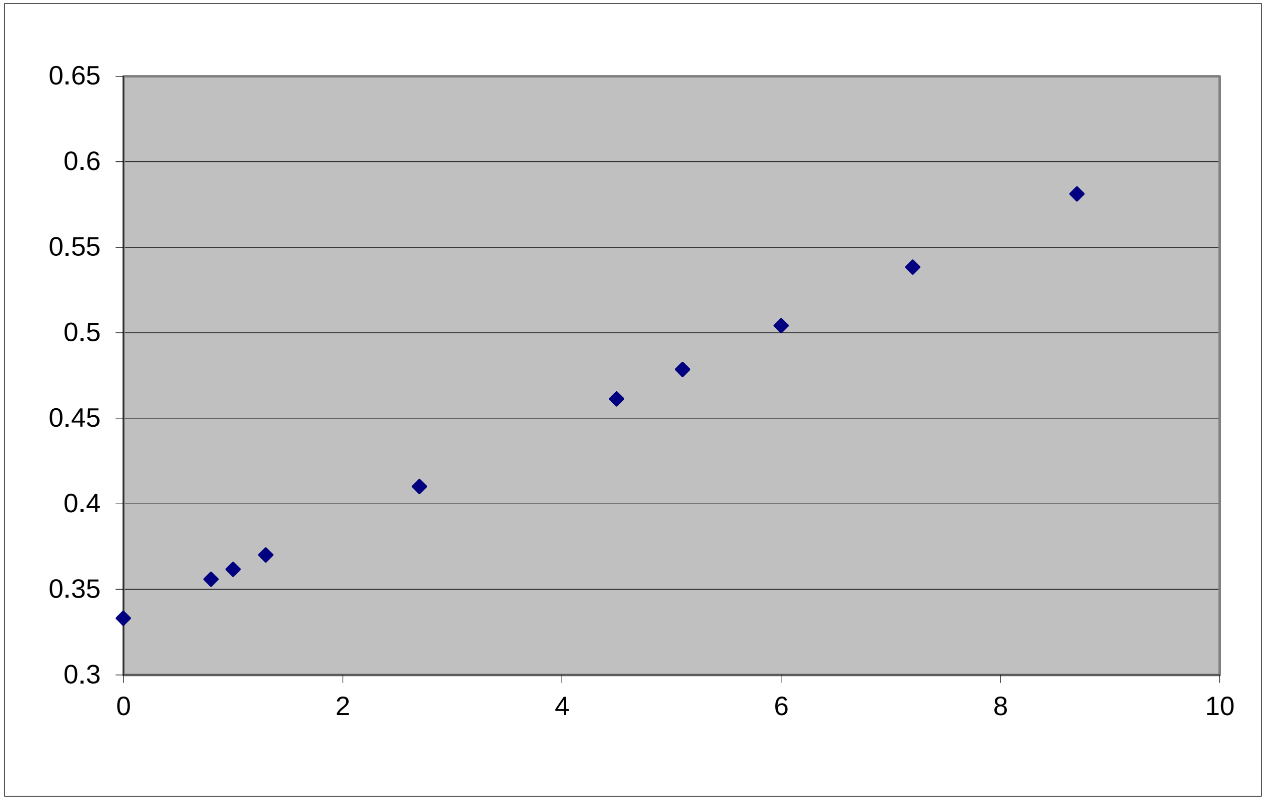
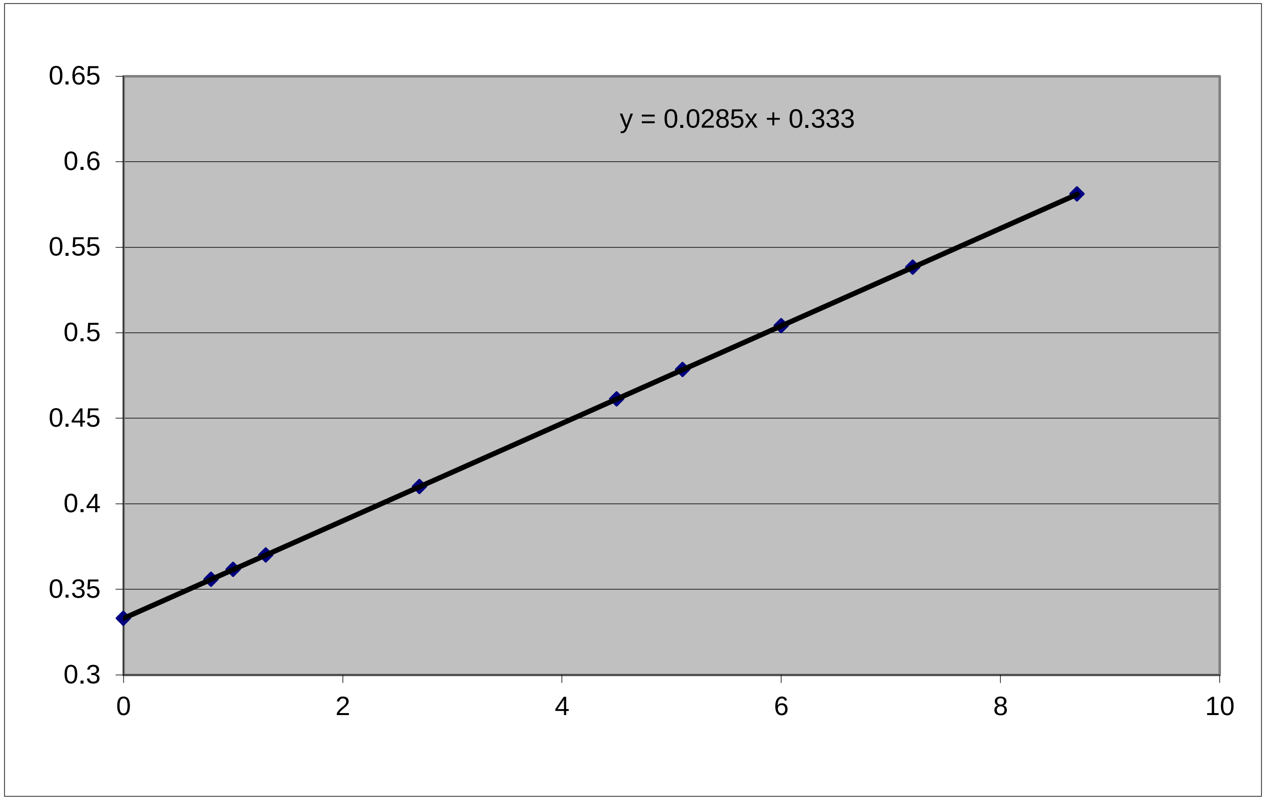
Measures of misfit – L1 and L2 norms



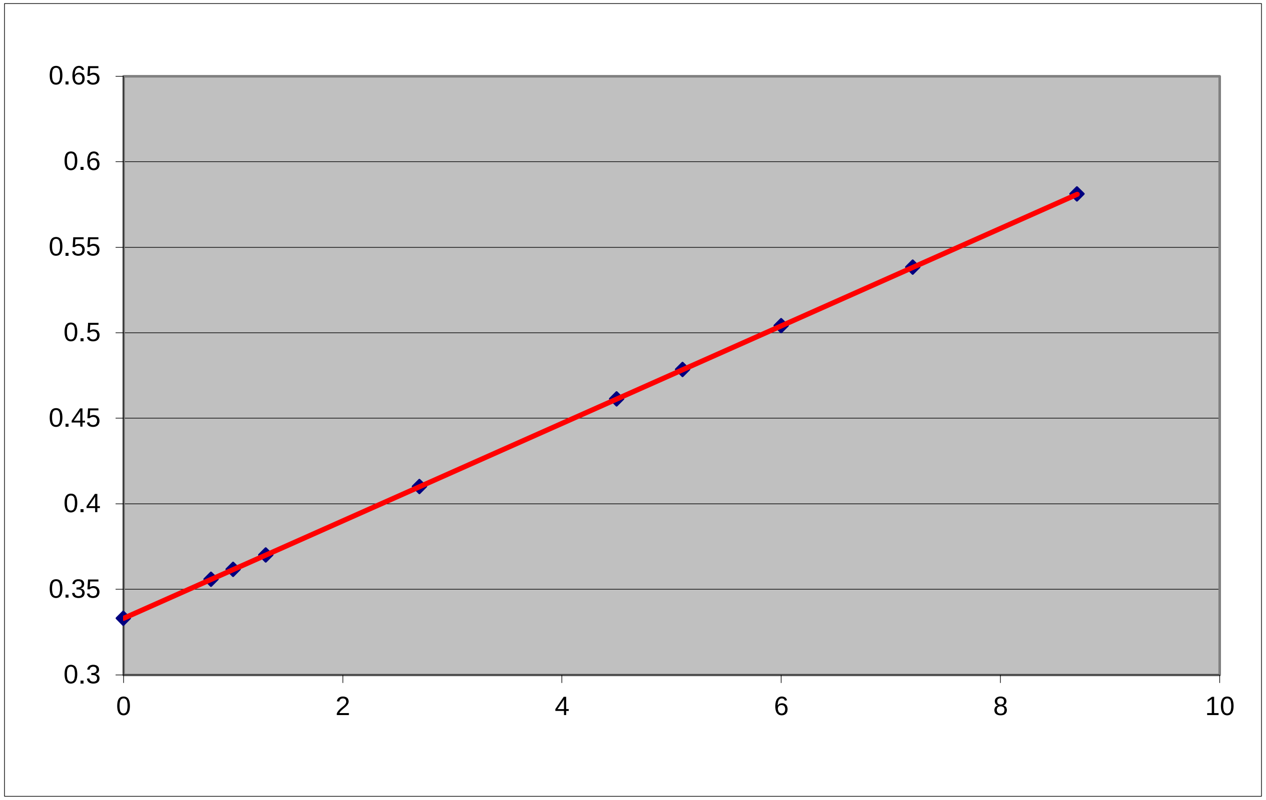
Measures of misfit – L1 and L2 norms



Original data

Misfit = sum of squares of residuals (L2 norm = least-squares)

Measures of misfit – L1 and L2 norms



Original data

Misfit = sum of absolute values of residuals (L1 norm)

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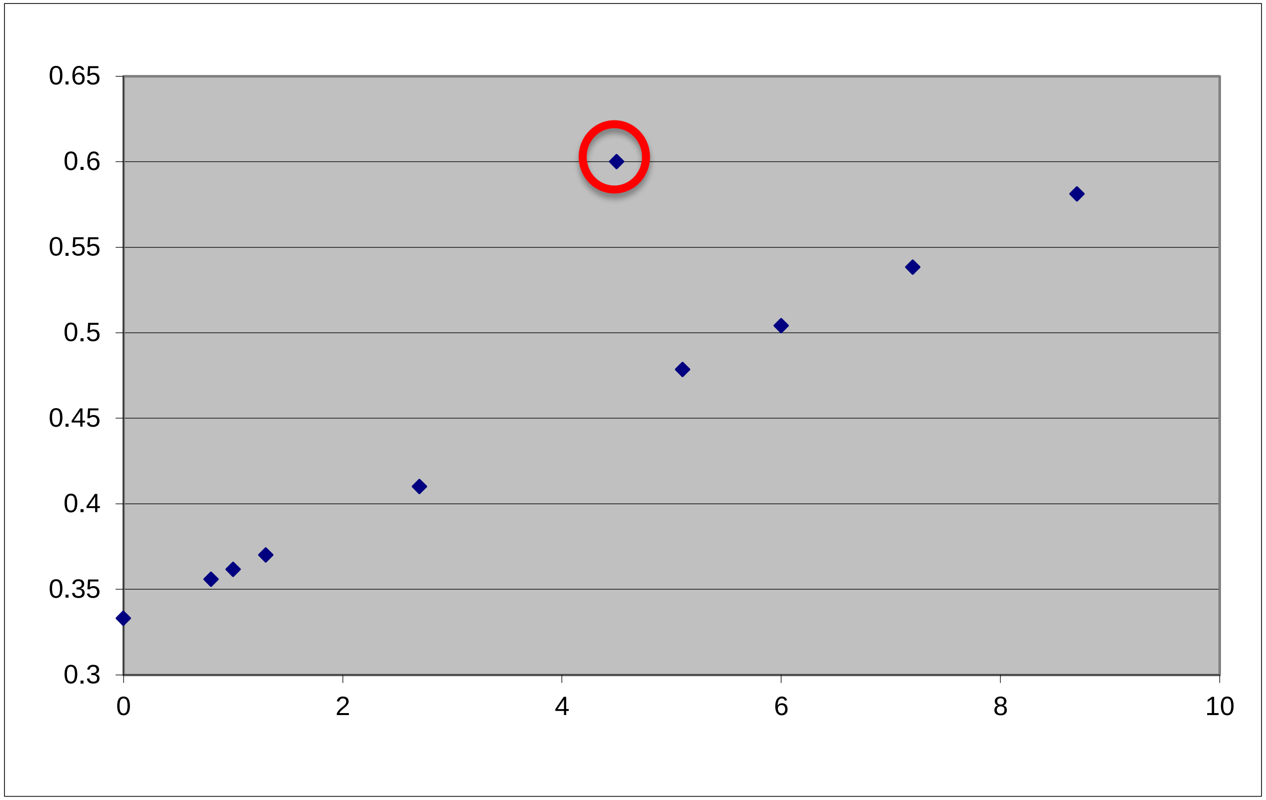
Measures of misfit – L1 and L2 norms



Add an outlying data point

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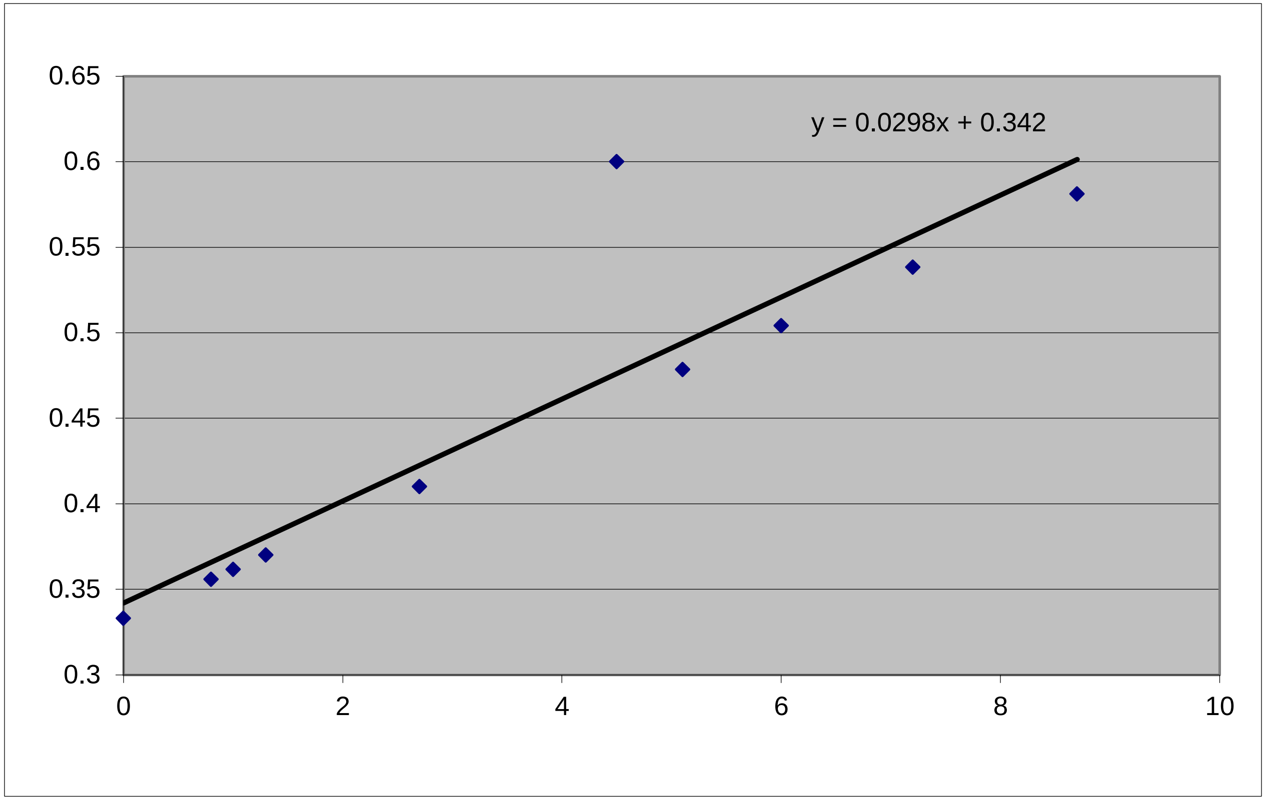
Measures of misfit – L1 and L2 norms



One outlying data point

Misfit = sum of squares of residuals (L2 norm = least-squares)

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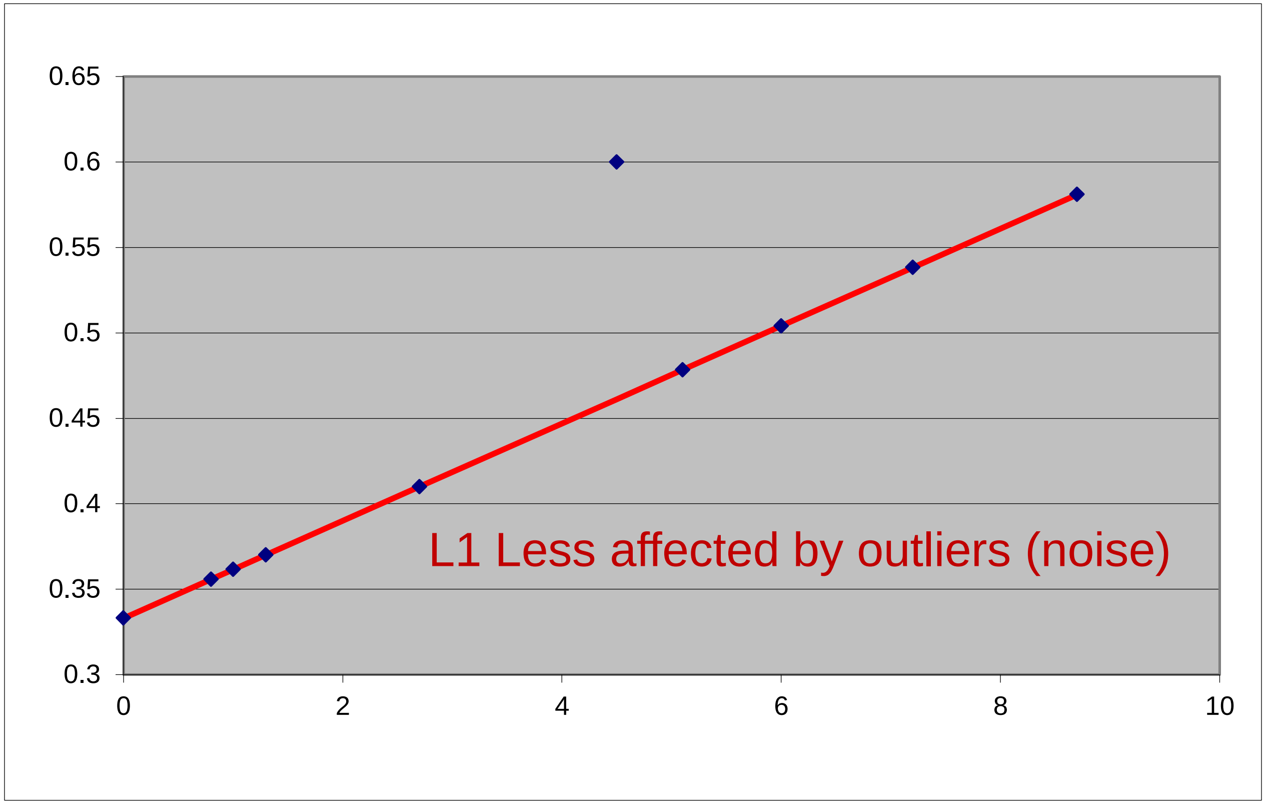
Measures of misfit – L1 and L2 norms



One outlying data point

Misfit = sum of absolute values of residuals (L1 norm)

Combining model norms and misfit

A statement of the inverse problem is:

Find the model which

Minimises the model norm (ϕ_M), and

Produces an acceptably small data misfit (ϕ_D)

Mathematically, this becomes a single optimisation

"Minimise $\phi = \phi_D + \beta\, \phi_M$ " (subject to ϕ_d < tolerance)

ϕ is the combined objective function

β is the trade off parameter (regularisation parameter)

β is the regularization parameter

- Solve:

  $\phi(m) = \phi_d(m) + \beta\,\phi_m(m)$

- β too large $\Rightarrow$ underfitting the data.

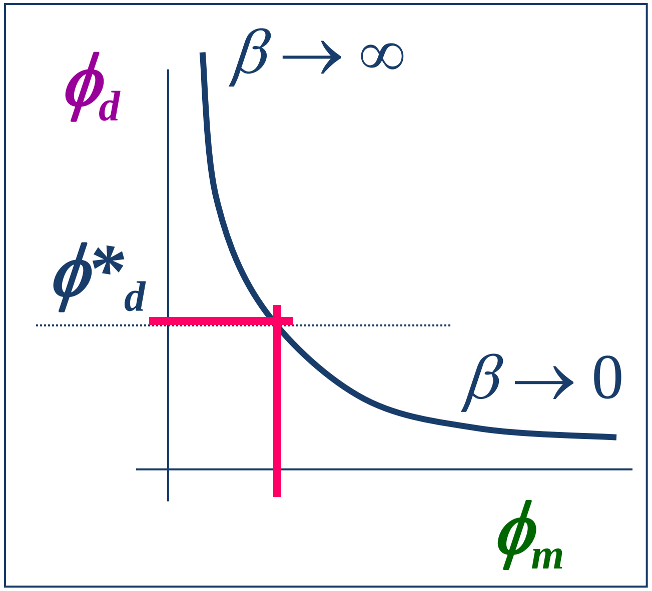
  Structural information lost.

- β too small $\Rightarrow$ overfitting the data.

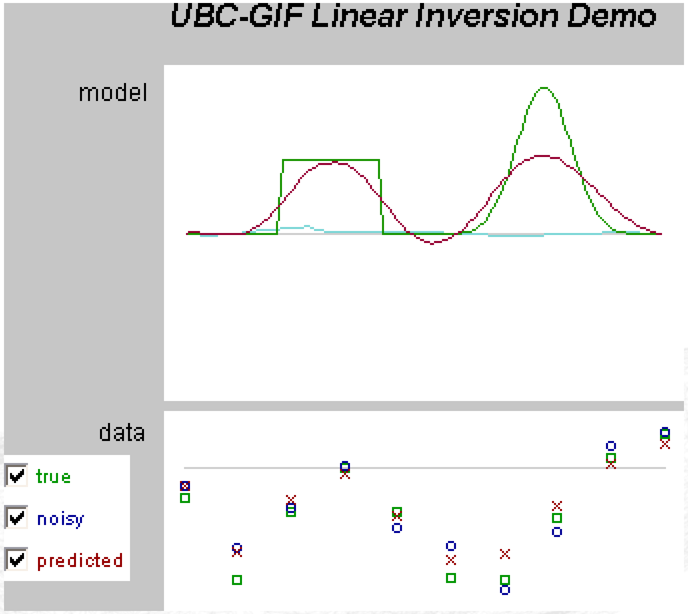
  Noise becomes imaged as structure.

- β just right ($\phi_d \approx N$) $\Rightarrow$ optimal fit.

  Best estimate of a model which adequately re-creates the observations.



Tikhonov curve



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Inverse Modelling

Non-Uniqueness: *Solution (partial…)*

- Provide explicit geological information
  - Constraints

- Combine information from independent geophysical methods
  - Joint or Cooperative Inversions

    e.g. Gravity with Magnetics, Airborne EM with CSAMT, etc.

Integrated Modelling: Constraints

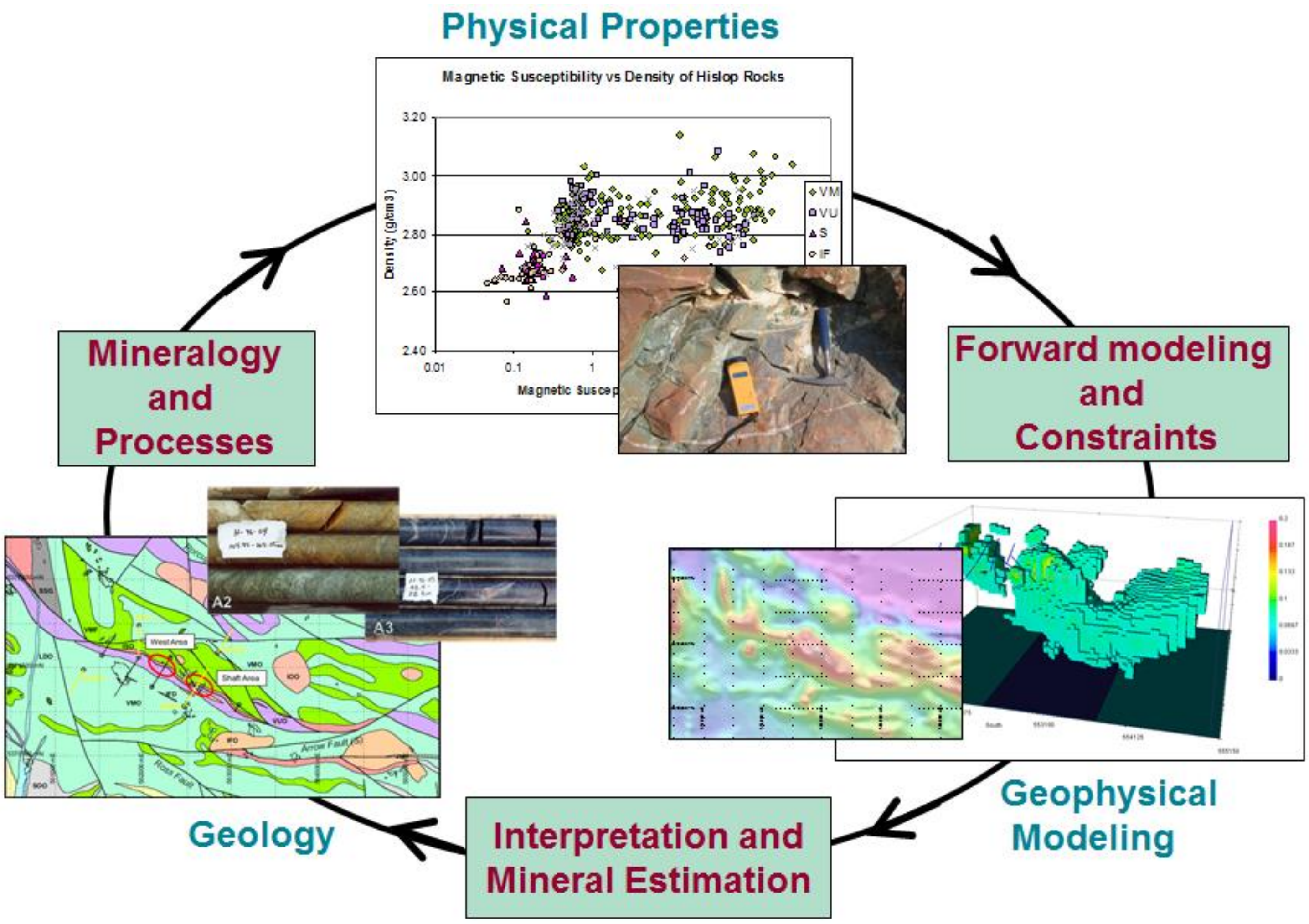
Sources of Data

- Geologic Mapping
- DH geological logs
- Interpreted cross-sections
- 3D geological models
- Physical property data per lithology
- Located physical property data measurements

Some information is subjective and some information is objective.

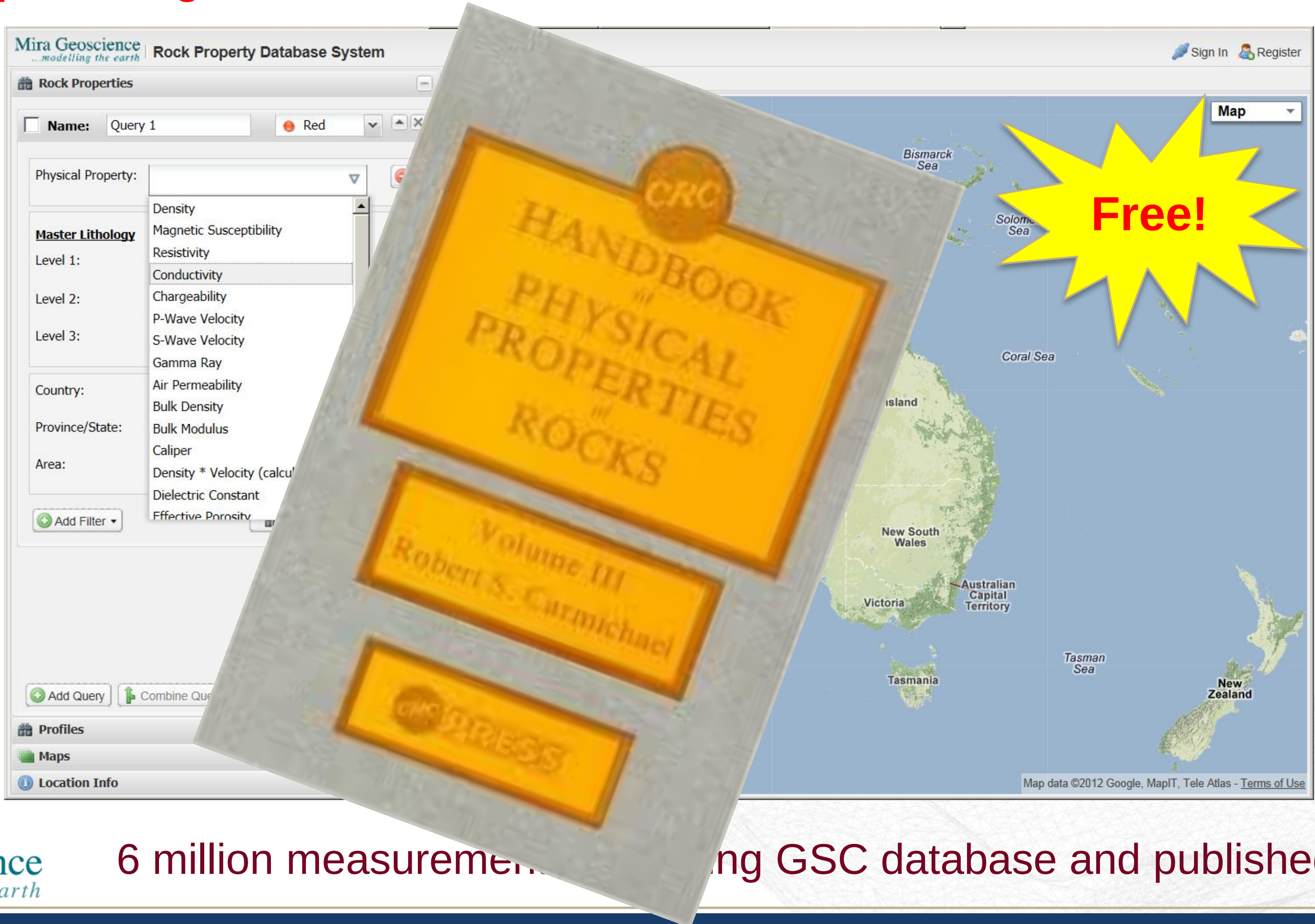
As with the geophysical data we would desire to quantify the uncertainty associated with this data as an input to the inversion.

Rock properties are the link between geology and geophysics



Shameless plug – Mira Geoscience **Rock Property Database System**

http://rpds.mirageoscience.com/



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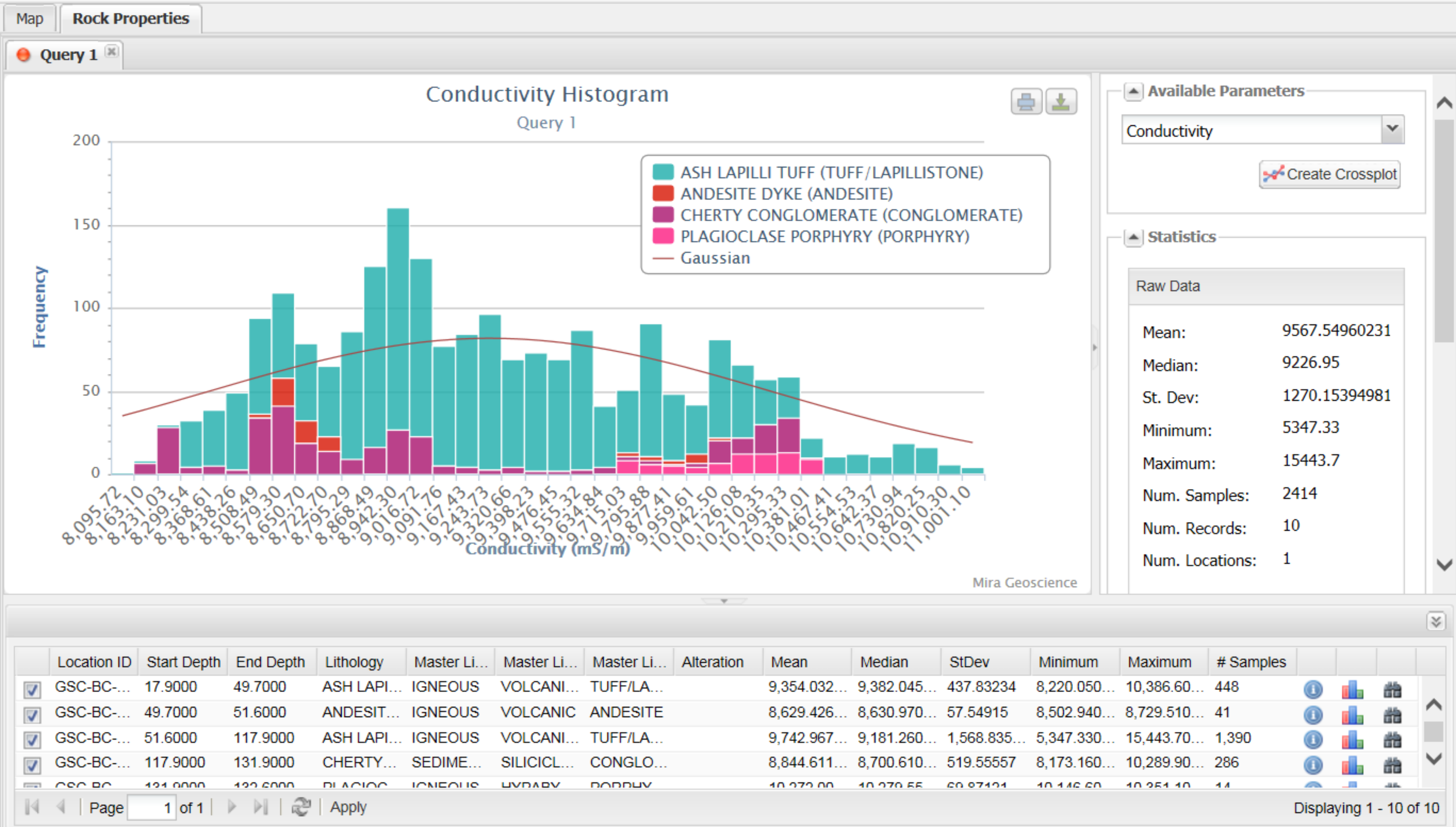
6 million measureme[...]ng GSC database and published data

Rock Property Database System



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Organise, understand, preserve and provide access to physical property data

Common Earth Modelling

Extending the model to include multiple properties, that honour multiple data sets, on a single model object.

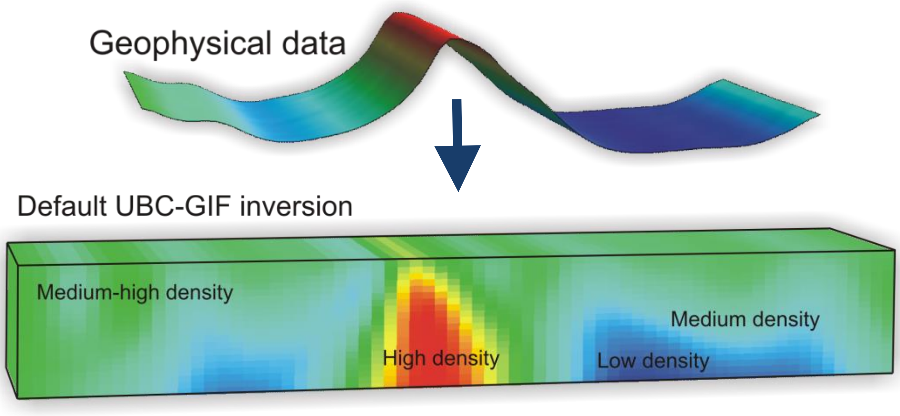
Goal:
Obtain the most complete representation of the earth.

Benefits:
Improved resolution away from constraints

Allows more precise exploration using quantitative 3D GIS analysis.

Common Earth Modelling: Constrained Inversion Modelling

2D Gravity Synthetic



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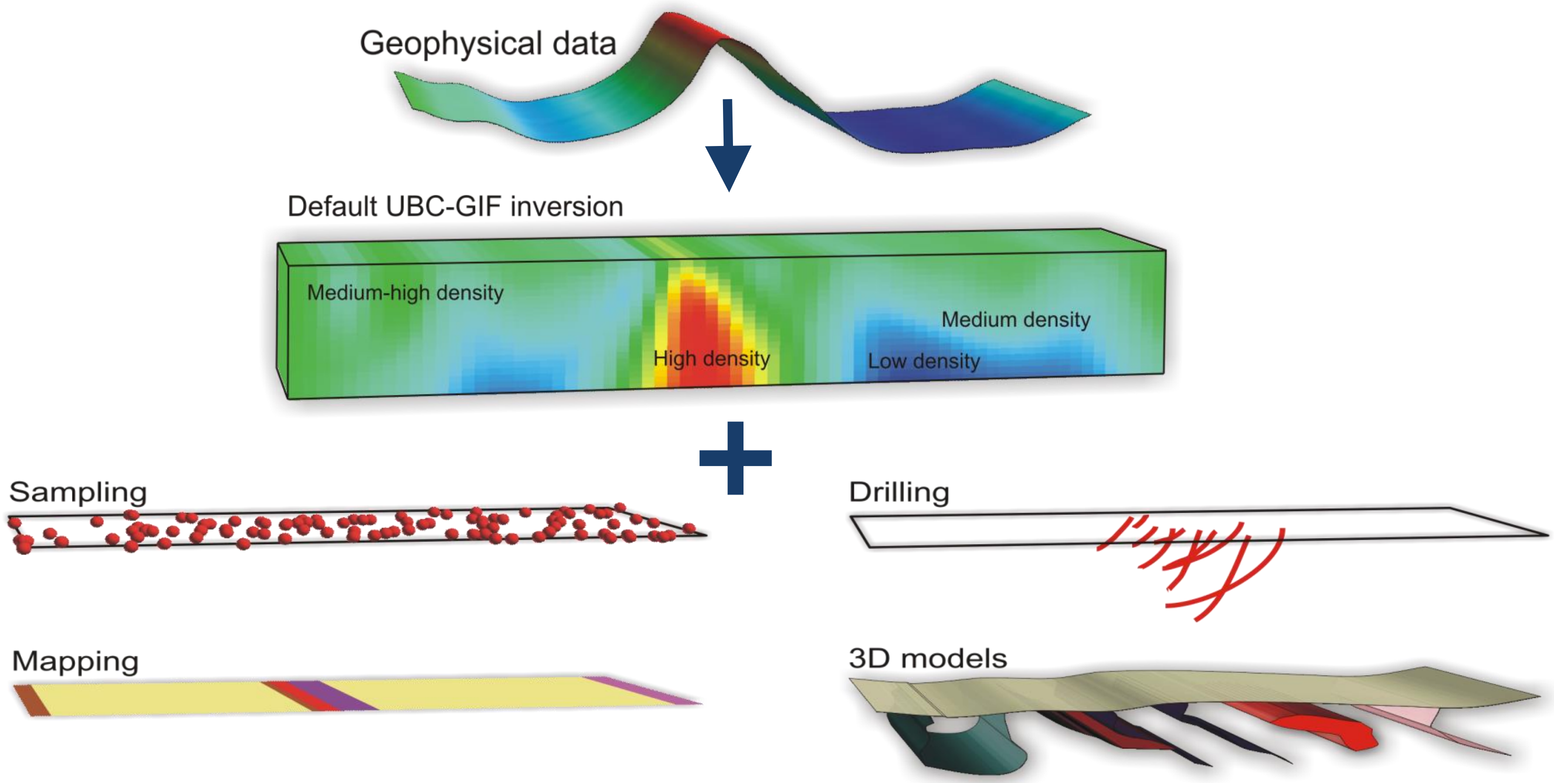
(Nick Williams)

Common Earth Modelling: Constrained Inversion Modelling

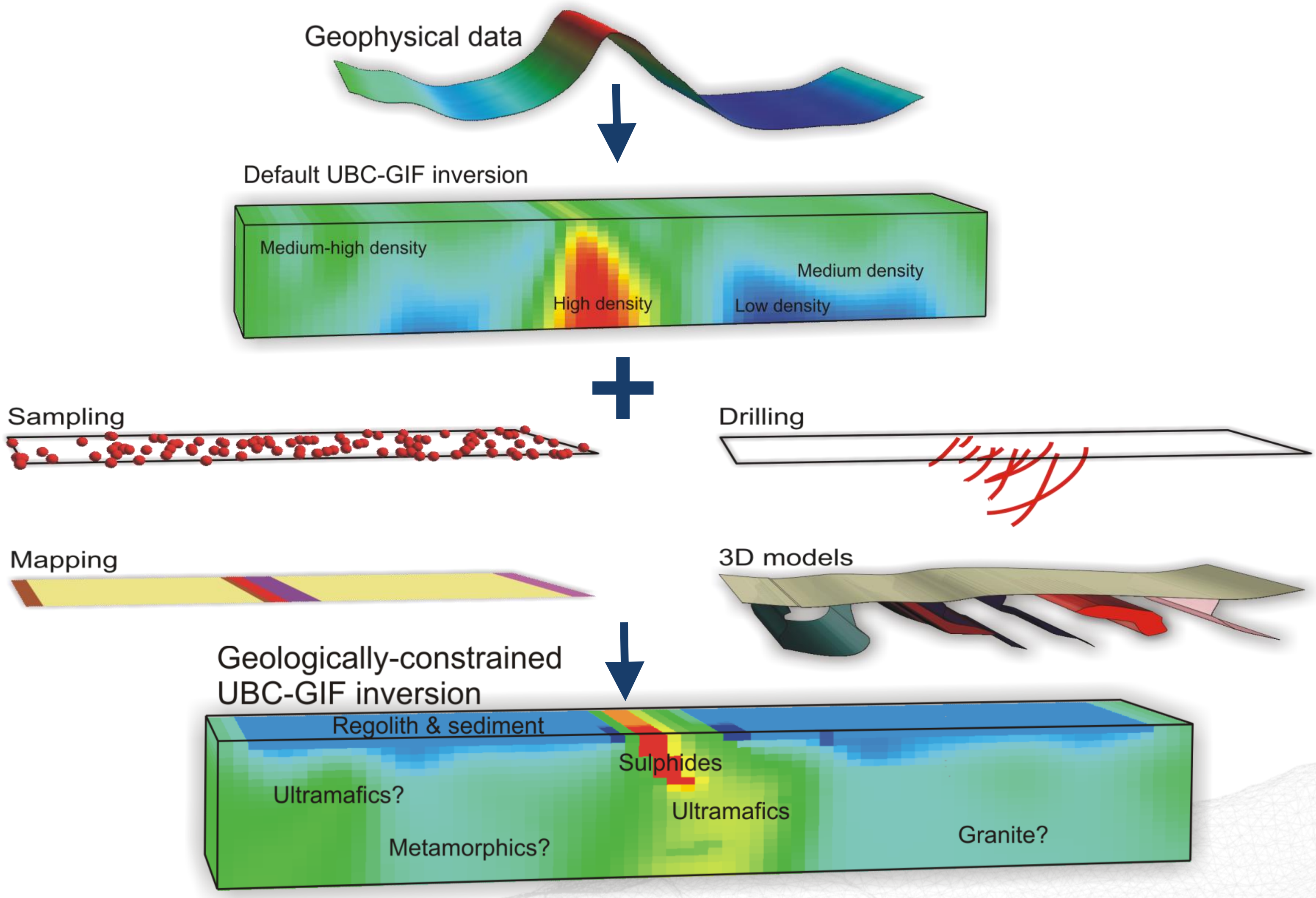
2D Gravity Synthetic



(Nick Williams)

Common Earth Modelling: Constrained Inversion Modelling

2D Gravity Synthetic



(Nick Williams)

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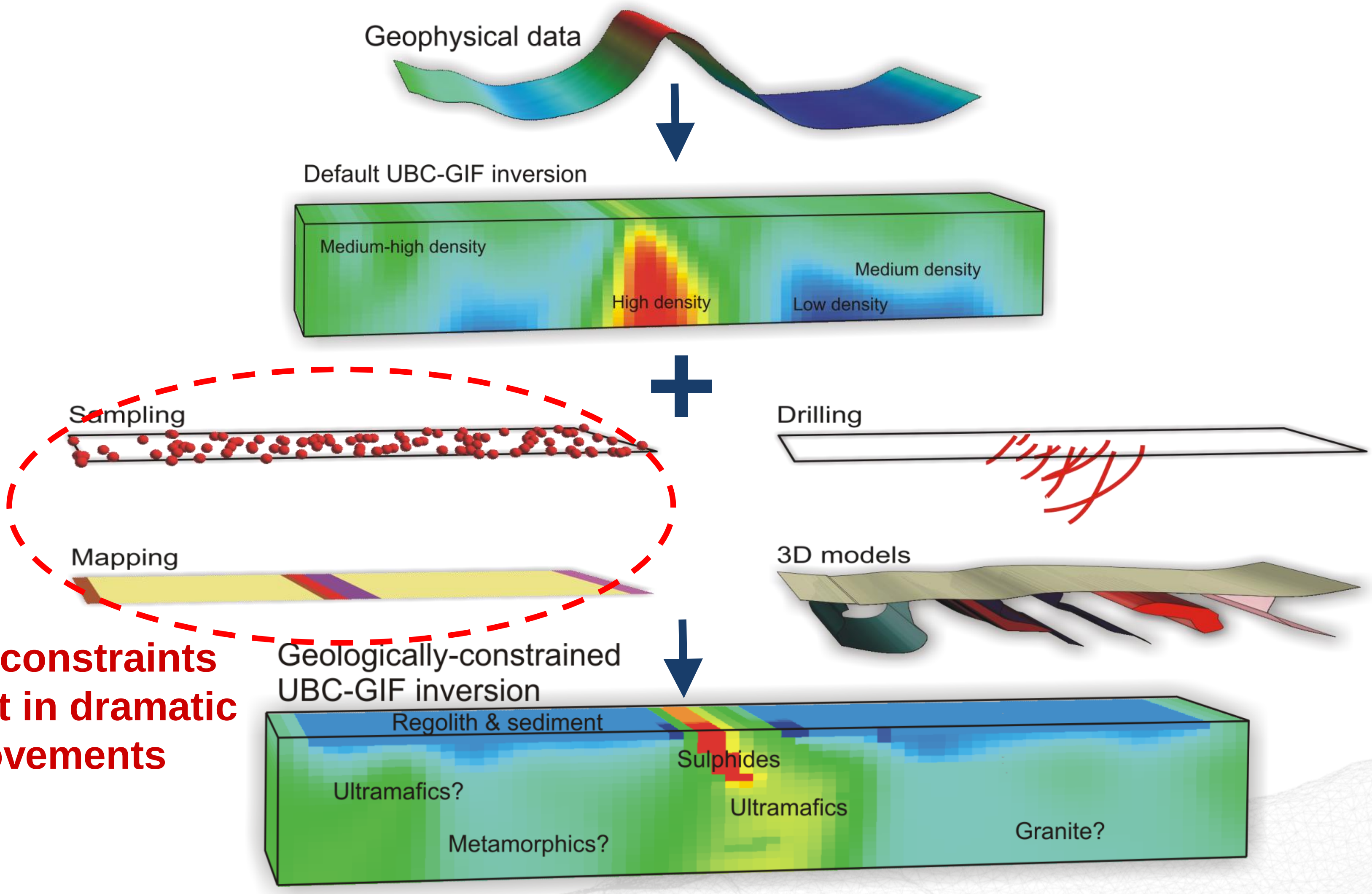
Common Earth Modelling: Constrained Inversion Modelling

2D Gravity Synthetic



Surface constraints can result in dramatic improvements

(Nick Williams)

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Joint and cooperative inversion

Inversion using more than one geophysical method

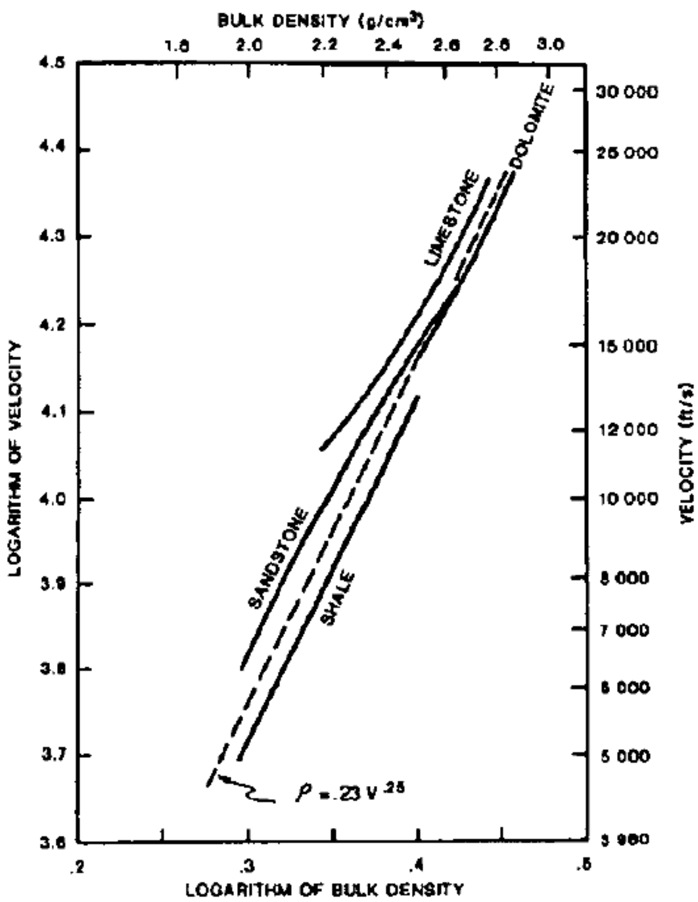
- Methods sensitive to same physical property (e.g., TEM and CSAMT)

- Methods sensitive to related properties (e.g., seismic and gravity)

Joint inversion – single objective function

Cooperative inversion – iterative/sequential approach

These approaches require that we establish relationships between the physical properties each method is sensitive to



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Appraisal – How good is our model?

Over-fitting vs under-fitting data

Limits to the data

Limits to the physics

Depth of investigation

Suite of models

Point-spread functions

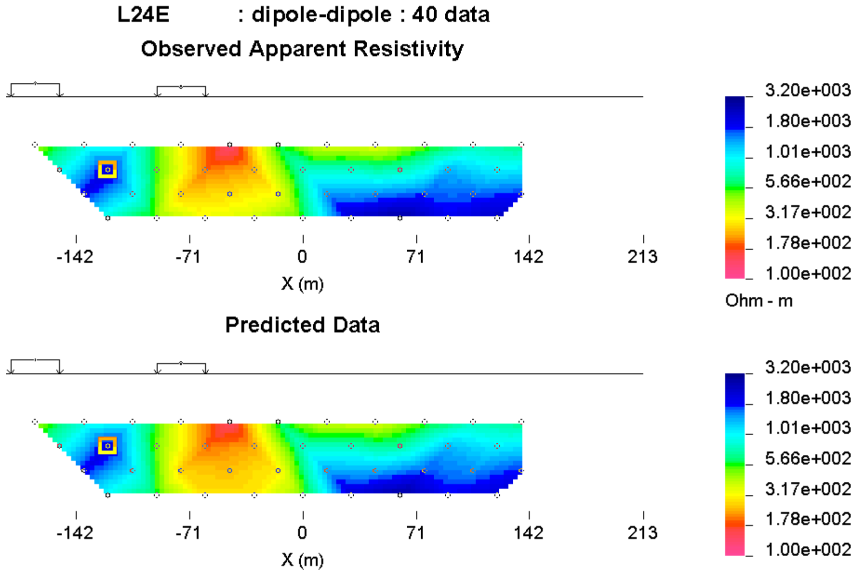
Model resolution analysis

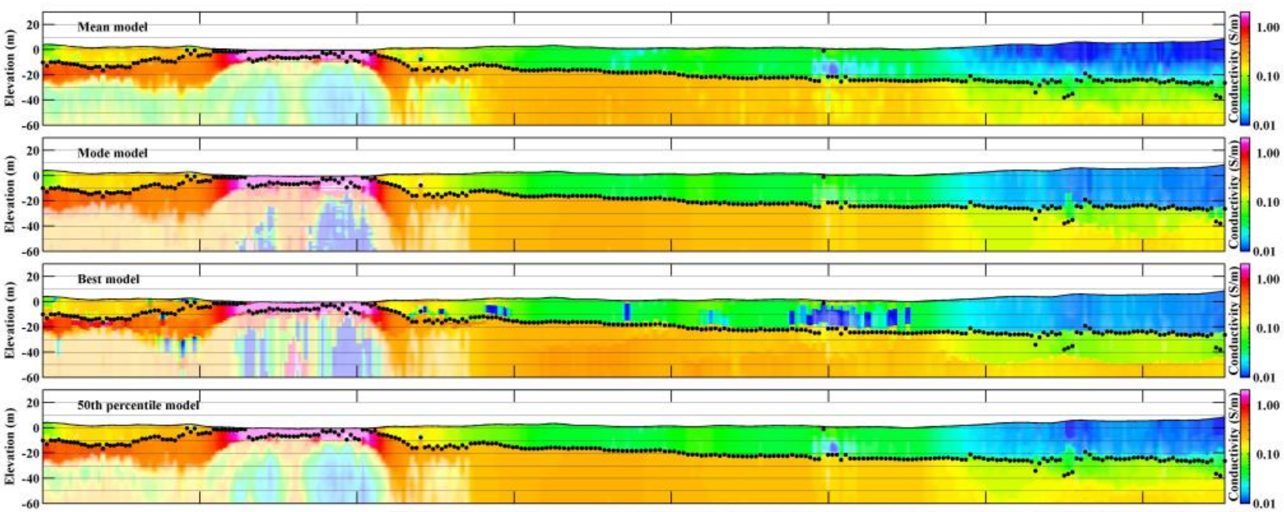
Sensitivity analysis

Extremal models

Model Covariance Matrix

Co-Kriging error





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Summary and conclusion

Inversion has the potential to greatly improve the geological interpretation of geophysical data

- High quality data is essential for the success of geophysical modelling
- More appropriate/efficient surveys can be designed
- Complex data sets can be understood (DH IP, 3D EM)

Understanding physical property data is the key to successful inversion interpretation.

- Rock type
- Alteration
- Mineralization

Summary and conclusion

Non uniqueness in inversion is dealt with by imposing constraints

- Provide the constraints or they will be provided for you
- Minimum structure or geological

Interpretation of inversion requires understanding of which parts of the model are driven by constraints and which parts are driven by data.

- Requires inspection of multiple models

Inspect observed and predicted data before accepting a model.

- Did the inversion fit the data anomalies you are interested in?
- Beware of over-fitting and under-fitting your data

Summary and conclusion

Geologically constrained inversion will greatly improve your results

- Constraints can be factual or conceptual (hypothesis testing)
- Sparse or detailed
- From different sources

  Geological maps

  Outcrop samples

  Estimates of overburden depth

  Detailed drill data

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Reference

Inversion for Applied Geophysics

http://www.eos.ubc.ca/research/ubcgif/iag/iag-outline.htm